MATERIAL FORCE METHOD FOR EVALUATING THE MATERIAL AND DISSIPATION FORCES FOR A CRACK IN AN INELASTIC BODY

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The objective of this work is to develop a method for calculating the Eshelbian driving force of a crack in an inelastic medium. It is assumed that the constitutive behavior of the inelastic medium conforms to an internal variable framework. The inelastic stress response is derived from a Helmholtz free energy function, and the evolution equations for the internal variables are associated with a positive dissipation potential. Given the solution of the direct motion problem for a cracked body, a corresponding boundary value problem for the inverse motion of the cracked configuration is posed. From the strong form of the new boundary value problem, an expression for the global material force is obtained. The expression consists of the contour integral of the Eshelbian-material tractions acting on the external boundary less the volume integral of the local dissipation forces in the regular part of the body. In a fracture mechanics context, this corresponds to an energy balance statement relating the applied energy release rate, G_{app}, to the energy release rate available at the crack tip for fracture, G_{tip}. The two quantities are equal in an elastic body, but in an inelastic body $G_{tip} \leq G_{app}$ as some of the applied energy can be lost to bulk dissipation. For numerical evaluation in a finite element framework, an equivalent domain integral expression for the material force is developed from the weak form of the boundary value problem. An appropriate Galerkin discretization is applied to the domain integral expression. This yields an approximation for the global material and dissipation forces that is equal to the sum of corresponding nodal quantities. Careful consideration is given to the discretization method. Particularly, the issue of computing quantities that depend upon strain gradients is addressed. Several competing choices will be contrasted here. It is expected that if the crack tip fields are resolved well by the finite element solution, the only appreciable value of the nodal material force lies at the crack tip node. However, the nodal dissipation forces form a vector field that can be used to visualize the crack-tip dissipation zone. The method is used to calculate the global material and dissipation forces for a cohesive crack growing in an applied K-field. The simulations are performed for a linear viscoelastic and J_2 elasto-plastic body. The results show that the global material force captures well the cohesive energy. Moreover, the sum of the global material and dissipation forces reproduces the energy release rate of the applied K-field, thus validating the proposed numerical method.